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A QUASIFRONT IN THE PROBLEM OF THE ACTION OF AN INSTANTANEOUS POINT IMPULSE AT THE EDGE OF A CONICAL SHELL†

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The problem of the effect of a longitudinal point impulse at the edge of a thin conical shell is considered. The motion of the shell is described by two-dimensional equations corresponding to the second low-frequency long-wavelength approximation in the theory of elasticity. They are investigated using the matched asymptotic expansion method. The quasifront phenomenon in the theory of two-dimensional shells is treated as a boundary layer in the neighbourhood of a subcharacteristic.

It is well known that the classical theories of degenerate systems (rods, plates and shells) distort the velocity of the extension wave when compared with the three-dimensional theory of elasticity. In the neighbourhoods of the points (or lines) at which such theories predict discontinuities for the parameters of the stress–strain state, the exact solution of the problem has a sharply-defined extremum which preserves smoothness. Thus the extension wavefront from the classical theories actually turns out to be a quasifront.

It has been shown [1] that a satisfactory description of a quasifront in one-dimensional problems in the dynamics of shells of revolution (the ring-loading case) can be obtained using the equations of the so-called plane dynamical boundary layer, basically identical with the equations of the plane theory of elasticity at the meridional section of the shell (the case of a plane strain). An alternative approach to the investigation of a quasifront in shells of revolution was employed in [2], where it was proposed that in order to smooth the discontinuity at the quasifront one should use the high-order low-frequency long-wave approximations of the equations of the theory of elasticity. There it turned out that the transition from the classical equations of motion which are the lowest-order low-frequency long-wave approximation to the high-order approximation equations is associated only with corrections to inertial terms. (For more details see [3, 4].)

The approach used in [2] was extended in [5] to the two-dimensional problem of the action of a longitudinal point impulse at the edge of a semi-infinite plate (an extension of the plane Lamb problem). In this case the quasifront propagates from the source making along semicircles in the midplane of the plate.

This paper extends earlier work [5] by using the method of matched asymptotic expansions [6, 7] to consider the problem of the effect of an instantaneous point impulse at the edge of a semi-infinite truncated circular conical shell of thickness $2h$. It is assumed that the impulse is directed along the generator of the cone and is uniformly distributed across its thickness.

1. FUNDAMENTAL RELATIONS OF THE PROBLEM AND THEIR TRANSFORMATION

Let the position of an arbitrary point A along the midsurface of the shell Γ be defined by the distance α along the generator of the cone from the point to the boundary ∂T (which is taken to be a circle of radius R) and the angle θ between the axial planes passing through A and through a reference point (Fig. 1).

We will take the fundamental equations in the following form.

The equations of motion ($\mathbf{u} = u_\alpha(\alpha, \theta, t)\mathbf{i}_\alpha + u_\theta(\alpha, \theta, t)\mathbf{i}_\theta$)

$$Eh\left[\frac{1}{1+\nu}\Delta\mathbf{u} + \frac{1}{1-\nu}\operatorname{grad}\operatorname{div}\mathbf{u}\right] - 2\rho h\frac{\partial^2}{\partial t^2}[\mathbf{u} - h^2 D_0 \operatorname{grad}\operatorname{div}\mathbf{u}] = 0, \quad \left(D_0 = \frac{\nu^2}{3(1-\nu)^2}\right) \tag{1.1}$$

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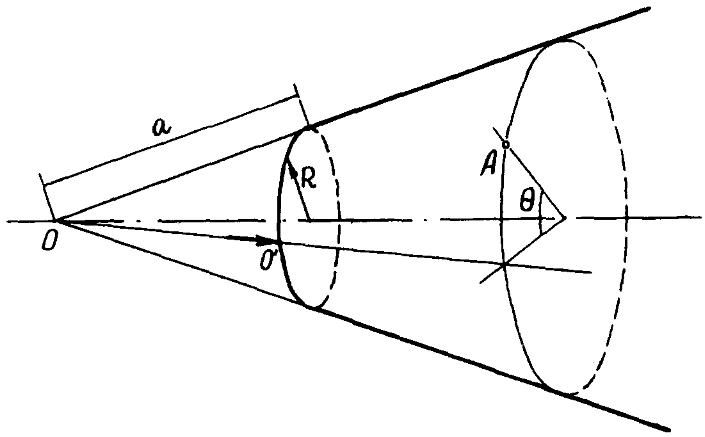

Fig. 1.

The "stress–displacement" formulae

$$T_{\alpha\alpha} = \frac{2Eh}{1-\nu^2}\left[\frac{\partial u_\alpha}{\partial \alpha} + \nu\left(\frac{1}{R(1+\alpha/a)}\frac{\partial u_\theta}{\partial\theta} + \frac{1}{a+\alpha}u_\alpha\right)\right]$$

$$T_{\alpha\theta} = \frac{Eh}{1+\nu}\left[\frac{1}{R(1+\alpha/a)}\frac{\partial u_\alpha}{\partial\theta} + \frac{\partial u_\theta}{\partial\alpha} - \frac{1}{a+\alpha}u_\theta\right] \tag{1.2}$$

and the boundary conditions ($\alpha = 0$)

$$T_{\alpha\alpha} = -P\delta(t)\delta(\theta), \quad T_{\alpha\theta} = 0 \tag{1.3}$$

Here $t \geqslant 0$ is the time, E is Young's modulus, ν is Poisson's ratio, ρ is the density of the shell material, $u_\alpha(\alpha, \theta, t)$, $u_\theta(\alpha, \theta, t)$ are the displacements of the midsurface Γ along the axes of the curvilinear system of coordinates (α, θ), $T_{\alpha\alpha}(\alpha, \theta, t)$, $T_{\alpha\theta}(\alpha, \theta, t)$ are the stresses at the midsurface, P is the amplitude of the force, δ is the delta function, Δ and the grad div are two-dimensional operators on Γ, and a is the distance along the generator from the vertex of the cone Γ to the end-section of radius R.

We take the initial conditions to be null.

The half-thickness of the shell is taken to be small compared to the radius R, i.e. $\eta = h/R \to 0$.

When $D_0 \equiv 0$ Eqs (1.1) are identical with the equations of the quasitangential vibrations of a shell (vibrations in which displacements within the midsurface dominate) that have been thoroughly investigated within the framework of the classical Kirchhoff–Love theory (see for example [8]). Except for the metric they are identical with the equations of the plane stressed state. The correction term with the factor D_0 introduced in [3, 4] "spoils" the hyperbolicity of these equations and, as will become clear later, enables one to smooth the discontinuity at the quasifront. Continuing the analogy with the plane problem in the theory of elasticity, one can treat Eqs (1.1) as the second asymptotic approximation of the three-dimensional equations of the theory of elasticity for the case of a plane stressed state [3, 4].

The domain of applicability of Eqs (1.1) is clearly limited by the bilateral inequality

$$\eta \ll l \ll 1 \tag{1.4}$$

imposed on l, which is the deformation wavelength expressed in terms of R. The upper limit of this inequality governs the zone in which one can, at a first approximation, ignore the effect of the shell curvature. The lower limit gives a coarse estimate of the domain of applicability of any two-dimensional shell theory as a long-wave approximation to the equations of the theory of elasticity. Better values for the lower limit in inequality (1.4) (taking into account the errors accumulating in the phase of the propagating vibration modes) were obtained in [3, 4, 9, 10], etc. for long-wave approximations in the theory of elasticity at various orders. With some additional restrictions the corresponding refinement of inequality (1.4) looks as follows:

$$\eta^{4/5} \ll l \ll 1 \tag{1.5}$$

We introduce potentials φ and ψ in the usual way

$$u_\alpha = \frac{\partial\psi}{\partial\alpha} - \frac{1}{R(1+\alpha/a)}\frac{\partial\varphi}{\partial\theta}, \quad u_\theta = \frac{\partial\psi}{\partial\alpha} + \frac{1}{R(1+\alpha/a)}\frac{\partial\varphi}{\partial\theta} \tag{1.6}$$

Substituting (1.6) into (1.1) the latter can be transformed to the form

$$\Delta\varphi - \frac{1}{c_3^2}\frac{\partial^2\varphi}{\partial t^2} + D_0\frac{h^2}{c_3^2}\frac{\partial^2\Delta\varphi}{\partial t^2} = 0 \quad \left(c_3^2 = \frac{E}{(1-\nu^2)\rho}\right) \tag{1.7}$$

$$\Delta\psi - \frac{1}{c_2^2}\frac{\partial^2\psi}{\partial t^2} = 0 \quad \left(c_2^2 = \frac{E}{2(1+\nu)\rho}\right) \tag{1.8}$$

Equation (1.7) describes the propagation of an extension wave and (1.8) that of a shear wave. We call φ and ψ the extension and shear wave potentials, respectively. It is clear from (1.7) and (1.8) that the refinement introduced in (1.1) only affects the extension wave and that the equations describing the shear wave remain hyperbolic. Analysis of the extension wave in the central topic of this paper.

The extension wave velocity c_3 corresponding to a degenerate ($D_0 \equiv 0$) hyperbolic Eq. (1.7) governs the previously mentioned quasifront. A priori, it is already clear that refinement introduced into the extension wave potential equation is only significant in the neighbourhood of the quasifront, where the asymptotically leading terms on the left-hand side of Eq. (1.7) (terms without the factor h^2) cancel themselves out.

Substituting representations (1.6) into (1.2) and then into (1.3), we rewrite the boundary conditions in terms of the potentials

$$\frac{Eh}{1+\nu}\left[\frac{2}{R}\frac{\partial^2\varphi}{\partial\alpha\partial\theta} - \frac{2}{Ra}\frac{\partial\varphi}{\partial\theta} + \frac{\partial^2\psi}{\partial\alpha^2} - \frac{1}{R^2}\frac{\partial^2\psi}{\partial\theta^2} - \frac{1}{a}\frac{\partial\psi}{\partial\alpha}\right] = 0$$

$$\frac{2Eh}{1-\nu^2}\left[\frac{\partial^2\varphi}{\partial\alpha^2} + \frac{\nu}{R^2}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{\nu}{a}\frac{\partial\varphi}{\partial\alpha} - \frac{1-\nu}{R}\frac{\partial^2\psi}{\partial\alpha\partial\theta} + \frac{1-\nu}{Ra}\frac{\partial\psi}{\partial\theta}\right] = -P\delta(t)\delta(\theta) \quad (\alpha = 0) \tag{1.9}$$

The original problem has thus been reduced to the solution of Eqs (1.7) and (1.8) with boundary conditions (1.9) and null initial data. We solve it by the method of matched asymptotic expansions [6, 7]. For the problem under consideration this method is implemented by constructing two types of expansion: a boundary layer expansion (for Eq. (1.7)) acting in the neighbourhood of the quasifront, and an outer expansion of problem (1.7)–(1.9) constructed outside that neighbourhood. It then turns out that there is a transition domain in which these expansions are matched, and that the main contribution throughout the asymptotic expansion obtained is made by wavelengths satisfying inequality (1.5).

2. THE BOUNDARY LAYER IN THE NEIGHBOURHOOD OF THE QUASIFRONT

We return to Eq. (1.7) and use the dimensionless variables

$$\alpha_1 = \alpha/R, \quad \theta_1 = \theta, \quad t_1 = tc_3/R \tag{2.1}$$

taking the characteristic linear dimension of the problem to be the radius R.

In dimensionless variables Eq. (1.7) takes the form

$$\Delta_1\varphi - \frac{\partial^2\varphi}{\partial t_1^2} + D_0\eta^2\frac{\partial^2\Delta_1\varphi}{\partial t_1^2} = 0 \tag{2.2}$$

$$\left(\Delta_1 = \frac{\partial^2}{\partial\alpha_1^2} + \frac{1}{a_1+\alpha_1}\frac{\partial}{\partial\alpha_1} + \frac{a_1^2}{(a_1+\alpha_1)^2}\frac{\partial^2}{\partial\theta_1^2},\quad a_1 = \frac{a}{R}\right)$$

We will use singular perturbation theory [6] to investigate Eq. (2.2).

Because of the symmetry of the problem we will confine ourselves to the domain $0 < \theta_1 < \pi$. Moreover, to fix our ideas, we will assume that

$$\alpha_1 < t_1,\quad t_1 \sim 1 \tag{2.3}$$

The second condition in (2.3) means that we only consider times at which the distance tc_3 over which the quasifront has travelled is comparable with the characteristic linear dimension R.

It is natural to use coordinates of the form

$$\zeta = a_1 \arccos\left[1 - \frac{t_1^2 - \alpha_1^2}{2a_1(a_1+\alpha_1)}\right] - \theta_1 \tag{2.4}$$

$$\beta = \arcsin\left[\frac{1}{2a_1 t_1}(2a_1\alpha_1 + \alpha_1^2 - t_1^2)\right],\quad \tau = t_1$$

when studying the boundary layer.

The physical meaning of the ζ and β coordinates is clear (Fig. 2). The midsurface Γ is shown as being rolled-out onto the plane. The dashed line represents the extension wave quasifront at time t_1, O' is the point of application of the load, A is the point under consideration on Γ with coordinates (α_1, θ_1, t_1), and M is the point of intersection of the quasifront line with the θ_1-line passing through A. The variable ζ is the length of the arc $A'M'$, i.e. the distance from the quasifront along the θ_1-line, normalized in a certain way ($\zeta > 0$ if the point under consideration on Γ is located behind the quasifront).

In a boundary layer domain one must assume that the derivative $\partial/\partial\zeta$ is large, so that one can introduce the boundary layer coordinate

$$\zeta_* = \zeta / \mu(\eta) \tag{2.5}$$

and a limiting process $\eta \to 0$ for fixed ζ_*, β and τ. The scaling factor $\mu(\eta)$ ($\mu(\eta \to 0)$ as $\eta \to 0$) describes the width of the boundary layer and will be subsequently determined from the requirement of asymptotic self-consistency in the resulting approximation equation.

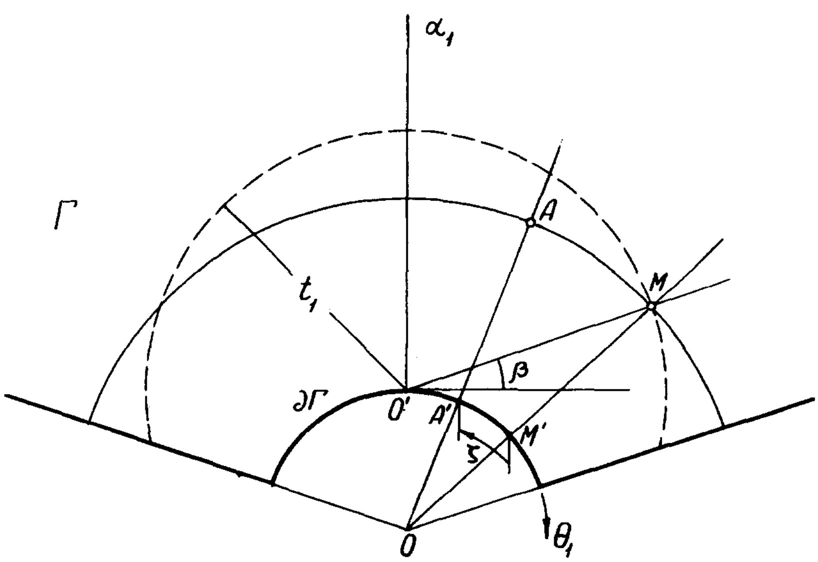

Fig. 2.

Changing from the system of coordinates $(\alpha_1, \theta_1, t_1)$ to (ζ_*, β, τ) and ignoring terms that are asymptotically secondary, we obtain the one-dimensional equation

$$\left(2\frac{\partial}{\partial\tau}+\frac{1}{\tau}\right)\varphi_{bl} - D_0\eta^2\frac{1}{\mu^3\cos^3\beta}\frac{\partial^3\varphi_{bl}}{\partial\zeta_*^3}=0 \tag{2.6}$$

for the leading term $\varphi_{bl}(\zeta_*, \beta, \tau)$ in the boundary layer expansion. This equation describes, at the first approximation, the boundary layer about the subcharacteristic (or characteristic in the corresponding degenerate equation) of Eq. (2.2). We call it the dynamical boundary layer surrounding the quasifront.

Requiring that the derivative with respect to ζ_* should enter into the asymptotically leading part of this equation, we obtain the estimate

$$\mu(\eta)=\eta^{2/3} \tag{2.7}$$

for the width of the boundary layer.

Note that by an appropriate choice of coordinates in similar problems for a cylindrical shell and a plate the equation for the boundary layer around a quasifront is identical with Eq. (2.6) up to asymptotically secondary terms. In these cases (see Fig. 3 for a plate) we have

$$\zeta=\sqrt{t_1^2-\alpha_1^2}-\theta_1,\quad \beta=\arcsin(\alpha_1/t_1) \tag{2.8}$$

Here, for a cylindrical shell $\alpha_1 = \alpha/R$, $\theta_1 = \theta$, where (α, θ) are cylindrical coordinates on Γ, and R is the radius of the midsurface, while for a plate $\alpha_1 = x/R$, $\theta_1 = y/R$, where (x, y) are Cartesian coordinates, while for the characteristic linear dimension R one can, for example, choose the distance travelled by the quasifront at the time under consideration.

We will represent the solution of Eq. (2.6) in the form of a Fourier integral. For convenience in subsequent matching with the outer solution we write it in the (ζ, β, τ) system

$$\varphi_{bl}=\int_{-\infty}^{+\infty}\Phi_0(k,\beta,\tau)\exp(ik\zeta)dk \tag{2.9}$$

$$\Phi_0(k,\beta,\tau)=\Phi_*(k,\beta)\frac{1}{\sqrt{\tau}}\exp\left[-i\frac{D_0\eta^2k^3}{2\cos^3\beta}\tau\right]$$

Here $\Phi_*(k, \beta)$ is an as yet unknown function. It will be determined when matching the boundary layer with the outer solution of the original problem.

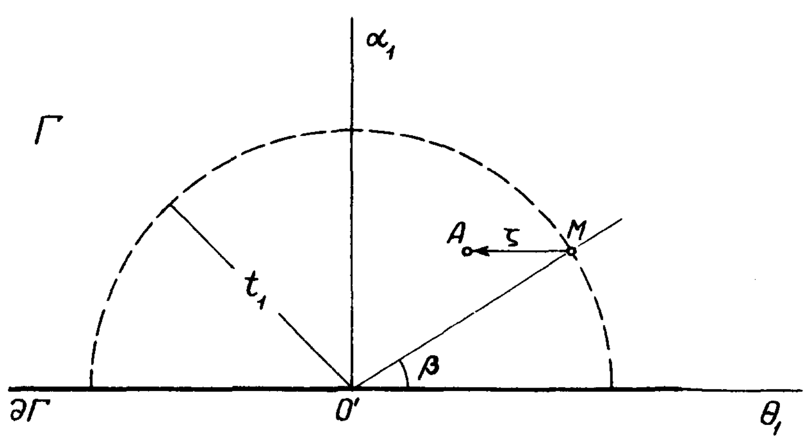

Fig. 3.

Before constructing the outer solution and the matching procedure, we once again stress that our main attention is focused on constructing the solution in the neighbourhood of the extension wave quasifront ($|\zeta| \ll 1$). Hence the outer expansion is only of interest for determining the unknown function $\Phi_*(k, \beta)$. An asymptotic analysis similar to that performed in [11] (see also [5]) shows that in the neighbourhood of the quasifront the main contribution to the integral (2.9) is from large values of the Fourier transformation parameter k. Below we shall assume, without any further explanation, that $|k| \gg 1$.

3. THE OUTER SOLUTION AND MATCHING PROCEDURE

We return to Eq. (2.2) written in (α_1, θ_1, t_1) coordinates, and consider the asymptotic process $\eta \to 0$ for fixed α_1, θ_1, t_1. It is obvious that the leading term $\varphi_o(\alpha_1, \theta_1, t_1)$ in the outer expansion (suitable at a certain distance from the quasifront) satisfies the limiting equation

$$\Delta_1 \varphi_o - \partial^2 \varphi_o / \partial t_1^2 = 0 \tag{3.1}$$

To determine φ_o we have the system of equations (3.1), (1.8) and (1.9). (We shall assume that all these equations are written in dimensionless coordinates.) A similar problem for a half-space has been investigated in detail in [11]. By analogy with [11] we shall seek a solution in the form of Fourier–Mellin integrals

$$\varphi_o(\alpha_1, \theta_1, t_1) = \int_{-\infty}^{+\infty} \Phi(\alpha_1, k, t_1) \exp(ik\theta_1) dk \tag{3.2}$$

$$\psi(\alpha_1, \theta_1, t_1) = \int_{-\infty}^{+\infty} \Psi(\alpha_1, k, t_1) \exp(ik\theta_1) dk$$

$$\Phi(\alpha_1, k, t_1) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} X(\alpha_1, k, s) \exp(st_1) ds \tag{3.3}$$

$$\Psi(\alpha_1, k, t_1) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} Y(\alpha_1, k, s) \exp(st_1) ds$$

Here $\sigma > 0$ and s is the Laplace transform parameter.

We obtain for the functions $X(\alpha_1, k, s)$, $Y(\alpha_1, k, s)$ a system of ordinary differential equations with variable coefficients

$$\frac{\partial^2 X}{\partial \alpha_1^2} + \frac{1}{a_1 + \alpha_1} \frac{\partial X}{\partial \alpha_1} - k^2 \left[\frac{a_1^2}{(a_1 + \alpha_1)^2} + \xi^2 \right] X = 0 \tag{3.4}$$

$$\frac{\partial^2 Y}{\partial \alpha_1^2} + \frac{1}{a_1 + \alpha_1} \frac{\partial Y}{\partial \alpha_1} - k^2 \left[\frac{a_1^2}{(a_1 + \alpha_1)^2} + \gamma^2 \xi^2 \right] Y = 0 \tag{3.5}$$

and boundary conditions ($\alpha_1 = 0$)

$$2ik \frac{\partial X}{\partial \alpha_1} - 2ik \frac{1}{a_1} X + \frac{\partial^2 Y}{\partial \alpha_1^2} + k^2 Y - \frac{1}{a_1} \frac{\partial Y}{\partial \alpha_1} = 0 \tag{3.6}$$

$$\frac{\partial^2 X}{\partial \alpha_1^2} + \frac{\nu}{a_1} \frac{\partial X}{\partial \alpha_1} - \nu k^2 X - (1 - \nu) ik \frac{\partial Y}{\partial \alpha_1} + (1 - \nu) ik \frac{1}{a_1} Y = -P_* \tag{3.7}$$

The general solution of Eqs (3.4) and (3.5) can be represented in exponential form

$$X(\alpha_1,k,\xi) = C_1(\alpha_1,k,\xi)\exp\left\{|k|\int_0^{\alpha_1}\chi_1(\alpha_1')d\alpha_1'\right\} + C(\alpha_1,k,\xi)\exp\left\{-|k|\int_0^{\alpha_1}\chi(\alpha_1')d\alpha_1'\right\}$$

$$Y(\alpha_1,k,\xi) = B_1(\alpha_1,k,\xi)\exp\left\{|k|\int_0^{\alpha_1}g_1(\alpha_1')d\alpha_1'\right\} + B(\alpha_1,k,\xi)\exp\left\{-|k|\int_0^{\alpha_1}g(\alpha_1')d\alpha_1'\right\} \quad (3.8)$$

It is assumed that any function f: $\{\chi_1, \chi, g_1, g\}$ satisfies the condition $\int_0^{\alpha_1} f(\alpha_1')d\alpha_1' \geqslant 0$ when $\alpha_1 > 0$. Using the null initial conditions we obtain the equations $C_1 = B_1 = 0$. The functions $\chi(\alpha_1)$ and $g(\alpha_1)$, and also the dependence of $C(\alpha_1, k, \xi)$ and $B(\alpha_1, k, \xi)$ on α_1 is determined by substituting (3.8) into Eqs (3.4) and (3.5). Here we again recall that for the matching procedure it is sufficient merely to construct the Fourier-image of the outer solution of the problem for large values of the parameter $|k|$. As a result we have

$$X(\alpha_1,k,\xi) = C_*(k,\xi)[1+\lambda^2(\alpha_1,\xi)]^{-1/4}\exp[-|k|p(\alpha_1,\xi)] \quad (3.9)$$

$$Y(\alpha_1,k,\xi) = B_*(k,\xi)[1+\lambda^2(\alpha_1,\gamma\xi)]^{-1/4}\exp[-|k|p(\alpha_1,\gamma\xi)]$$

$$p(\alpha_1,\xi) = a_1\left\{\sqrt{1+\lambda^2(\alpha_1,\xi)} - \sqrt{1+\xi^2} - \operatorname{arsh}\frac{1}{\lambda(\alpha_1,\xi)} + \operatorname{arsh}\frac{1}{\xi}\right\}$$

$$\lambda(\alpha_1,\xi) = (1+\alpha_1/a_1)\xi$$

We find the functions C_* and B_* after substituting (3.9) into boundary conditions (3.6) and (3.7). When $|k| \gg 1$ we have, at a leading term

$$C_* = -\frac{2P_*}{(1-\nu)k^2}F(\xi)(1+\xi^2)^{1/4}, \quad B_* = -\frac{2P_* i \operatorname{sign} k}{(1-\nu)k^2}G(\xi)(1+\xi^2\gamma^2)^{1/4} \quad (3.10)$$

$$F(\xi) = \frac{2+\xi^2\gamma^2}{r(\xi)}, \quad G(\xi) = \frac{2\sqrt{1+\xi^2}}{r(\xi)}, \quad r(\xi) = (2+\xi^2\gamma^2)^2 - 4\sqrt{1+\xi^2}\sqrt{1+\xi^2\gamma^2}$$

Substituting representation (3.9) into the Mellin integrals (3.3), and using (3.10) we obtain

$$\Phi(\alpha_1,k,t_1) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{C_*|k|}{[1+\lambda^2(\alpha_1,\xi)]^{1/4}}e^{-|k|[p(\alpha_1,\xi)-\xi t_1]}d\xi \quad (3.11)$$

$$\Psi(\alpha_1,k,t_1) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{B_*|k|}{[1+\lambda^2(\alpha_1,\gamma\xi)]^{1/4}}e^{-|k|[p(\alpha_1,\gamma\xi)-\xi t_1]}d\xi$$

Bearing in mind the symmetry of C_* and the antisymmetry of B_* with respect to k in (3.10), one can reduce the right-hand sides of formulae (3.2) to integrals over the positive semi-axis $0 < k < +\infty$. As a result we obtain

$$\varphi_o(\alpha_1,\theta_1,t_1) = -2\gamma^2 P_*\int_0^\infty J(\alpha_1,k,t_1)\frac{\cos(k\theta_1)}{k}dk \quad (3.12)$$

$$J(\alpha_1,k,t_1) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{(1+\xi^2)^{1/4}F(\xi)}{[1+\lambda^2(\alpha_1,\xi)]^{1/4}}e^{-|k|[p(\alpha_1,\xi)-\xi t_1]}d\xi$$

for the potential φ_o. (Similar expressions for the potential ψ are not given because its contribution in a neighbourhood of the quasifront is zero.)

The integral expressing the function J when $a_1 \to \infty$ (the conical shell degenerating into a cylinder) is identical with a thoroughly investigated integral from non-stationary dynamics [11, 12], and when $k \to \infty$ it is determined by a similar method of steepest descent. Omitting intermediate calculations, we give the final result

$$J = \frac{\sin\beta}{\sqrt{k\pi t_1}\cos^{3/2}\beta} F\left(\frac{i}{\cos\beta}\right) \mathrm{Re}\left\{(1+i)\exp\left[ika_1 \arcsin\left(\frac{t_1\cos\beta}{a_1+\alpha_1}\right)\right]\right\} \tag{3.13}$$

To match expansions (2.9) and (3.12) we apply the matching principle. This amounts to the outer expansion of the inner (i.e. boundary layer) expansion coinciding with the inner (boundary layer) expansion of the outer expansion. If this rule is satisfied, the expansions have overlapping domains of applicability.

Expanding (2.9) when $\eta \to 0$ with fixed α_1, θ_1, t_1, and (3.12) when $\eta \to 0$ with fixed ζ_*, β, τ, and comparing, we obtain the required function

$$\Phi_*(k,\beta) = -\frac{\gamma^2 P_*}{2\sqrt{\pi}} \frac{\sin\beta}{\cos^{3/2}\beta} F\left(\frac{i}{\cos\beta}\right) \frac{(1+i)}{|k|^{3/2}\sqrt{\operatorname{sign} k}} \tag{3.14}$$

Using (1.6) we finally obtain for displacements in the neighbourhood of the quasifront (in ζ, β, τ coordinates)

$$u_\theta = -\frac{\sqrt{2}\gamma^2 P_*}{\sqrt{\pi\tau}R} \frac{a_1}{\sqrt{a_1^2+\tau^2+2a_1\tau\sin\beta}} \frac{\sin\beta}{\cos^{3/2}\beta} F\left(\frac{i}{\cos\beta}\right) I$$

$$u_\alpha = \frac{\sin\beta + \tau/a_1}{\cos\beta} u_\theta \tag{3.15}$$

$$I = \int_0^{+\infty} \sin\left[k\zeta - \frac{D_0\eta^2 k^3\tau}{2\cos^3\beta} + \frac{\pi}{4}\right] \frac{dk}{\sqrt{k}} \tag{3.16}$$

If we allow for the different meanings of the variables (cf. (2.4) and (2.8)), integral (3.16) is identical with the similar integral for the plate problem [5]. All the conclusions in [5] that apply to integral (3.16) also apply here. Hence, in particular, at a distance $|\zeta| \ll \zeta_0 \ll 1$ ($\zeta_0 \sim \eta^{2/5}$) from the quasifront the main contribution is made by values of the Fourier transformation parameter k which correspond to monochromatic waves in the interval (1.5).

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